Arrangement of Elements 7th order & Element Sequences, Periodic Table
– Olivier Joseph –

7th order / 120 elements
3rd Heptale angle

H(n) = \frac{1}{12} \{ 2n^3 + 6n^2 + (7 - 3(-1)^n) n - 3(-1)^n - 9 \}

n \geq 1, n \in \mathbb{N}
9th order / 220 elements
3rd Heptale angle
[Extrapolation MA Lvl 1]

Figure 2: Spiral arrangement extended to 220 elements
Z = The Atomic Number, the Number of Protons in the Nucleus

\[ y = 0.2082x \]
\[ R^2 = 0.996 \]

**Figure 3** Atomic mass vs. Atomic number

Z = The Atomic Number, the Number of Protons in the Nucleus

\[ H(n) = \frac{1}{12} \left( 2n^3 + 6n^2 + (7 - 3(-1)^n) n - 3(-1)^n - 9 \right) \]
\[ n \geq 1 \]
\[ H(2p) = \frac{1}{6} \left( 8p^3 + 12p^2 + 4p - 6 \right) \quad \text{Even numbers} \quad p \geq 1 \]
\[ H(2p + 1) = \frac{1}{6} \left( 8p^3 + 24p^2 + 28p + 6 \right) \quad \text{Odd numbers} \quad p \geq 0 \]

\[ A(n) = \frac{1}{12} \left( 2n^3 + 6n^2 + (7 - 3(-1)^n) n - 3(-1)^n - 21 \right) \]
\[ n \geq 1 \]
\[ A(2p) = \frac{2}{7} \left( 2p^3 + 3p^2 + p - 3 \right) \quad \text{Even numbers} \quad p \geq 1 \]
\[ A(2p + 1) = \frac{2}{7} \left( 2p^3 + 6p + 7 \right) \quad \text{Odd numbers} \quad p \geq 0 \]

\[ K(n) = A(n)/2 \]
\[ n \geq 1 \]
\[ K(2p) = \frac{A(2p)}{2} \quad \text{Even numbers} \quad p \geq 1 \]
\[ K(2p + 1) = \frac{A(2p + 1)}{2} \quad \text{Odd numbers} \quad p \geq 0 \]

\[ K(2p + 1) = K(2p) + (p + 1)^3 \]
\[ H(2p + 1) = H(2p) + 2(p + 1)^3 \]

**Table 1** H(n) sequence (Group IA) elements (H, Li, Na, K, Rb, Cs,...and beyond), and sub-sequences

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<th>H(n)</th>
<th>A(n)</th>
<th>K(n)</th>
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OJ - G Sequences – sequences of elements

Table 2

Periodic table elements allocation and its presumed adequate extension

OJ - G Triangle
Figures 1, 2 and 4, 5 show that the elements are notably arranged in a specific pattern (in a spiral form made up of 7 arms) when an angle approximately equal to $\approx 102$ degrees is set. It is to be noted that other angle values generate various interesting patterns with different, but less remarkable distribution of elements.

With this specific angle of about 102 degrees, all the elements of a given group (corresponding to chemical elements with similar properties, such as Ne, Ar, Kr, Xe, Rn for the noble gases...) also become specifically distributed among each arm of the spiral in a notable arranged way, fairly close to a regular spiral pattern as well, curved in the opposite direction.
PHI Theor. Deviation 220 elements

3rd Heptale angle $\phi$

Matching / Ratio: $\epsilon$

Slope: $\log(\text{PHI})$

$\text{PHI} = \frac{1 + \sqrt{5}}{2}$

9th order / 220 elements

3rd Heptale angle $\phi$

[Extrapolation MA Lvl 1]

Figure 6

LogPHI vs. MA/MA$_{c12}$ - Matching comparison
Figures 6 and 7 exhibit another feature of the arrangement with the very close match observed between the two arrangements, consisting of the MA/MA_{c12} pattern as radial distance on one side and the LogPHI pattern (PHI = golden mean $\frac{1 + \sqrt{5}}{2}$) as radial distance on the other side. As depicted here below, both models appear to fairly overlap.

**Figure 7** LogPHI (in upper position and blue color) vs. MA/MA_{c12} overlapped patterns, showing very minor deviations.

**Figure 8** First five elements positioning, according to the constant angle $\gamma = \frac{\text{spacing}}{2}$ and MA/MA_{c12} as the radial distance
**Best $\gamma$ angle and associated $\xi$ ratio determination**

Given $\gamma$ angle, between two successive elements, characterized as follows, together with its associated $\xi$ ratio:

\[
A + B = 2\pi r \\
B = \gamma r \\
\gamma_{Rad} = \gamma_{Deg} \cdot \frac{\pi}{180} = \frac{2\pi B}{A + B} \\
\xi = \frac{A}{B} \quad \text{(called $\xi$ ratio)}
\]

\[
\gamma_{Rad} = \frac{2\pi}{\xi + 1} \\
\gamma_{Deg} = \frac{360}{\xi + 1}
\]

leading to:

\[
\xi = \frac{2\pi - \gamma_{Rad}}{\gamma_{Rad}} \\
\xi = \frac{360 - \gamma_{Deg}}{\gamma_{Deg}}
\]

Among the pool of possible angle values, the best candidate angle $\gamma_{Best}$ would presently be an angle that generates at least 7 spiral arms (close to a multiple of $2\pi/7$ in theory) and providing the most possible optimized distribution of elements. Such a preferred angle is an irrational angle.

The angle of $\pi$ rotations or a derived multiple of $\pi$ (i.e. $k \pi * 360^\circ$) matches the requirements.

\[
\gamma_{Best} = 2 \pi * 360
\]

This angle shall moreover be adequately expressed, by isolating its most valuable part with its remainder in a range of $0^\circ$ to $360^\circ$ (360 as the divisor). Therefore, $\gamma_{Best}$ is equivalently evaluated to $\gamma_{Deg}$:

\[
\gamma_{Best} = 2 \pi * 360 = \gamma_{Deg} + 6 \cdot 360 \quad \text{($\gamma_{Deg} = \text{Modulo} \ [\gamma_{Best}, 360^\circ]$)}
\]

\[
\gamma_{Deg} = 2 (\pi - 3) \cdot 360 \\
\gamma_{Rad} = 4 \pi (\pi - 3) \\
\xi = \frac{7 - 2\pi}{2(\pi - 3)}
\]

\[
\gamma_{Deg} \approx 101.9467106^\circ \approx 102^\circ \\
\gamma_{Rad} \approx 1.779305761 \\
\xi \approx 2.531256653
\]

\[
\begin{align*}
-102^\circ & \quad 28.3 \% & 2.531256653 & = \xi_1 = \frac{7 - 2\pi}{2(\pi - 3)} \\
-258^\circ & \quad 71.7 \% & 0.39506069 & = \xi_2 = \frac{2(\pi - 3)}{7 - 2\pi}
\end{align*}
\]
Finding the second degree polynomial equation having its 2 roots as \( \mathcal{X}_1 \) and \( \mathcal{X}_2 \), in the form of an equation and resulting from the equivalency between ratios.

\[
\begin{align*}
\mathcal{X}^2 - n\mathcal{X} - m &= 0 \\
\frac{A}{B} &= \frac{n A + m B}{A} \\
\mathcal{X} &= \frac{A}{B}
\end{align*}
\]

Equation solved by:

\[
\begin{align*}
x_1 &= \frac{n + \sqrt{n^2 + 4m}}{2} = \frac{7 - 2\pi}{2(\pi - 3)} = \mathcal{X}_1 \\
x_2 &= \frac{n - \sqrt{n^2 + 4m}}{2} = \frac{2(\pi - 3)}{2\pi - 7} = -\mathcal{X}_2 = -\frac{1}{\mathcal{X}_1}
\end{align*}
\]

\[\Rightarrow \quad \begin{cases} 
n = \frac{4\pi - 13}{2(\pi - 3)(2\pi - 7)} \\
m = 1
\end{cases}\]

\[
\mathcal{X}^2 - \frac{4\pi - 13}{2(\pi - 3)(2\pi - 7)} \mathcal{X} - 1 = 0
\]

In other words, this means:

\[
\mathcal{X}^2 = \Pi \mathcal{X} + 1 \quad \text{given} \quad \Pi = \frac{4\pi - 13}{2(\pi - 3)(2\pi - 7)} \approx 2.136195963
\]

Or which can be rearranged to

\[
\frac{1}{\mathcal{X}} = \mathcal{X} - \Pi
\]

Leading to \( \mathcal{X} \):

\[
\mathcal{X} = \Pi + \frac{1}{\Pi + \frac{1}{\Pi + \frac{1}{\Pi + \cdots}}}
\]
Approximation, Sequences associated to $\Phi$ :

Specifying the recursive sequences:

$$X_n = \Pi + \frac{1}{X_{n+1}}$$

$n \geq 1$

$X_0 = \Pi$

Alternatively, the below recurrence relation can be expressed and approximates $\Phi$ :

$$U_{n+2} = \Pi U_{n+1} + U_n$$

$n \geq 1$

$U_1 = 1$

$U_2 = \Pi$

$$\Phi = \lim_{n \to \infty} \left( \frac{U_{n+1}}{U_n} \right).$$

$\frac{U_{n+1}}{U_n}$ ratio properly converging to $\Phi_1$ after a few iterations as $n$ index increases
U(k) sequence according to k index (using Binet forms):

As previously stated, the 2 roots \( X_1 = \phi = \phi \) and \( X_2 = -1/\phi \) satisfy the equation \( X^2 = \Pi \phi + 1 \)

\[
X_1^2 = \Pi X_1 + 1 \\
X_2^2 = \Pi X_2 + 1
\]

Thus, the sequence \( F_k = \alpha \phi^k + \beta \left(-\frac{1}{\phi}\right)^k \) satisfies the same recurrence relation

\[
F_{k+2} = \Pi F_{k+1} + F_k
\]

Indeed,

\[
F_{k+2} = \alpha \phi^{k+2} + \beta \left(-\frac{1}{\phi}\right)^{k+2} = \alpha \Pi \phi^{k+1} + \alpha \phi^k + \beta \Pi \left(-\frac{1}{\phi}\right)^{k+1} + \beta \left(-\frac{1}{\phi}\right)^k = \Pi F_{k+1} + F_k
\]

\[
F_k = \alpha \phi^k + \beta \left(-\frac{1}{\phi}\right)^k
\]

\[
F_0 = \alpha + \beta \\
F_1 = \alpha \phi - \beta \frac{1}{\phi}
\]

Setting initial conditions:

\[
F_0 = 0 \\
F_1 = 1 \\
F_2 = \Pi
\]

It follows, after simplification:

\[
\alpha = -\beta
\]

\[
\alpha = \frac{2(\pi - 3)(7 - 2\pi)}{(8\pi^2 - 52\pi + 85)} \approx 0.341726437
\]

\( F_k \) can consequently be expressed, according to k index and \( \phi \) ratio as follows:

\[
F_k = U(k) = \frac{2(\pi - 3)(7 - 2\pi)}{(8\pi^2 - 52\pi + 85)} \left[ \phi^k - \left(-\frac{1}{\phi}\right)^k \right]
\]

where \( \phi = \frac{7 - 2\pi}{2(\pi - 3)} \)
Figure 8  Initial and final arrangement mock-up (conical helix)

\[ \gamma_{\text{Deg}} \approx 101.9467106^\circ \approx 02^\circ = \mathcal{S} \]
### Ж and Powers of Ж:

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Spiral arrangement in numbers:

The below figure shows the distribution of elements in spiral shape with the successive layers, elements names, and their atomic numbers.

Figure 9  Spiral arrangement of elements with their Atomic numbers and names
ANNEXE : Standard form of the periodic table
(two standard representations as grids of elements with the periods in rows and groups in columns, and used as the baseline for the calculations and definition of the above 7-arms spiral arrangement of elements).

![Tableau périodique des éléments](image-url)

![Tableau périodique des éléments chimiques](image-url)